**Lecture-02**

**Complex Differentiation and**

**The Cauchy-Riemann Equation**

**Analytic Functions:**

If a single valued function is differentiable i.e. exists at every point of a domain *D* except possibly at a finite number of exceptional points then the function is said to be **analytic** in the domain *D*. These exceptional point at which does not exist are called **singular points** or **singularities of the function**.

**Necessary conditions for to be analytic:**

**RECTANGULAR FORM:**

If and satisfies the **Cauchy-Riemann equations(C-R)** i.e.,

 and 

i.e.,  and 

then ****is said to be **analytic.**

Hence**,** at points whereexists may be obtained from either of

 or 

**POLAR FORM:**

If and satisfies the **Cauchy-Riemann equations(C-R)** i.e.

 and 

i.e.,  and 

then ****is said to be **analytic.**

Hence, at points whereexists may be obtained from either of

****

**Important Formulae:**

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |

**Example: 1**

Verify **C-R** equations for the function and hence find.

**Solution:** Given,

or,

or,

or,

or,

Here, and .

Now, partially differentiating and with respect to and, we get

From the above result, we can write

and

Since satisfies Cauchy-Riemann equations, so is analytic.

|  |
| --- |
| **Example: 2**  Verify **C-R** equations for the function and hence find . |
| **Solution:**  Given    or,  or,  or,  or,  Here , and  Partially differentiating and with respect to and, we get  From the above result, we can write  and .  Since satisfies Cauchy-Riemann equations, so is an analytic function.            . |

**Exercise set: 2.1**

1. Write Cauchy-Riemann (**C-R**) equations in rectangular and polar forms.
2. For the following functions:
3. separate real and imaginary parts,
4. verify **C-R**  equations,
5. find or .
6. Are the following functions analytic? If analytic, then find 



1. Are the following functions analytic? If analytic, then find 



Reference Book: Advanced Engineering Mathematics (10th edition) by Erwin Kreyszig, Herbert Kreyszig, Edward J. Norminton, published by John Wiley & Sons, Inc